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Bayesian Inversion-Driven Fairness Optimization for Dance Competitions

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Abstract: In order to solve the problems of complete lack of fan voting data and opaque interaction mechanism between judges and audiences in the program "Dancing with the Stars", this paper proposes a three-step algorithm framework. In the first step, a reverse fan inference model based on Bayesian sequential update and Monte Carlo constraint sampling is constructed, and the posterior distribution of fan votes is inverted by using the weekly judge scores and elimination results, and the reproduction accuracy of the model for historical elimination reaches 99.68%, and the estimation uncertainty is quantified. In the second step, a fan-judge composite ranking model is established, Spearman's correlation coefficient and rescue rate index are introduced, and the ranking system and the percentage system are compared, and it is found that the influence of fans and judges under the ranking system is balanced (correlation coefficient 0.88 vs 0.86), while the influence of fans is diluted under the percentage system (0.12), and the judge's life-saving mechanism is more effective in the ranking system. The third step is to propose a multi-dimensional influence model of rule stage perception, and to quantify the contribution of age, industry background, and professional dance partners to judges' scores and fan votes through Ling regression, and define the preference difference $\Delta\beta$ to reveal that the judges are biased towards technology (professional dance partner coefficient is up to 0.81), and fans are biased towards identity (industry background coefficient is up to 0.48), and the differences intensify with the evolution of the competition system. The algorithm framework in this paper provides quantitative decision support for program production.

Keywords: Bayesian sequential update; Reverse inference; constraint sampling; Ridge returns; Compound ranking.

1. Introduction

"Dancing with the Stars" (DWTS) is a reality show that combines professional dance and celebrity effects, and its elimination results are determined by the judges' on-site scoring and audience voting. However, viewer voting data was never made public, making it impossible to directly analyze fan preferences; At the same time, the program has undergone many rule adjustments (ranking system, percentage system, and judges' life-saving links), and the balance between the influence of judges and fans under different competition systems has always been the focus of controversy in academia and industry[1][2]. How to reverse hidden fan voting from observable judge scores and elimination results, quantify the impact of the competition system on fairness and entertainment, and analyze the heterogeneous effect of multiple factors (age, professional background, dance partner) on scoring is an important topic in the field of entertainment data analysis [3][4].

Existing studies have mostly focused on the aggregation of ratings in sports competitions [5] or the statistical modeling of audience voting in talent shows [6], but there is no mature backward thrust framework for scenarios where the judge-fan dual track system and fan data are completely missing. In the first step, an inverse fan inference model is proposed, which treats fan voting as a latent variable, uses the hard constraint of the elimination result for Monte Carlo sampling, and characterizes the time-varying inertia supported by fans through Bayesian sequential update [7]. This method not only reconstructs the posterior distribution of fan votes in each period, but also gives the estimated uncertainty interval, laying the foundation for subsequent analysis [8][9].

The second step focuses on the competition system comparison. The ranking system and the percentage system have completely different aggregation methods for the weight of judges and fans, which directly affects the elimination results [10][11]. This paper constructs a fan-judge composite ranking model, introduces bias indicators and rescue rates, and reveals that the probability of fans saving the lowest judges under the ranking system is as high as 60%, while under the percentage system, it is only 51%. The Judges' Saving Session (from Q28) resulted in 62.5% of eliminations being directly decided by the judges [13]. These findings quantify the leverage effect of format design [14].

The third step is to focus on the influence of individual characteristics. Are there systemic differences in the attractiveness of contestants' age, industry background, and professional partner ability to judges and fans? Existing literature suggests that judges pay more attention to technical execution [15], while fans are susceptible to celebrity aura [16]. In this paper, a multidimensional influence model of rule stage perception is established, and ridge regression is carried out in stages according to the ranking system, percentage system, and new rule, and the standardized coefficient makes cross-factor comparable. The results confirm that the judges' emphasis on professional dance partners is as high as 0.81 (ranking system), and the fan preference coefficient for industry background is 0.48. Further defining the preference difference $\Delta\beta$, it is found that the judges' preferences gradually converge towards technology, and the fan preferences are always bound to identity [17].

The data in this article comes from the official 34-season history, including weekly judge ratings (average) and elimination results [18]. In order to meet the requirements of the model, the data is standardized, and the following reasonable assumptions are made: the distribution of fan votes that can reproduce the elimination results is equally possible (inversion is unbiased); The systematic differences in

elimination results mainly stem from the design of the competition system. After controlling the competition stage, the influence of player characteristics on the score is almost linear. The relative importance of fairness and entertainment changes over the course of the season. Section 2 below details the three-step model establishment, Section 3 gives the solution and discussion of the results, and Section 4 summarizes.

2. Methods

2.1. Reverse fan inference model

This model is designed to reverse the distribution of weekly fan votes from known judge scores and elimination results. Suppose there are N_t contestants in the week t, the judges' scoring vector is $\mathbf{J}_t = (J_{t,1}, \ldots, J_{t,N_t})$, and the fan vote latent variable $\mathbf{F}_t = (F_{t,1}, \ldots, F_{t,N_t})$ satisfies the non-negative and sums are 1: $F_{t,i} \geq 0, \sum_{i=1}^{N_t} F_{t,i} = 1$. According to the program rules, there are two definitions of comprehensive score:

$$S_{t,i} = \tilde{J}_{t,i} + F_{t,i} \tag{1}$$

Under the percentage system, where $\tilde{J}_{t,i}$ is the standardized judge's score during the week; Under the ranking system, $S_{t,i} = \text{rank}(J_{t,i}) + \text{rank}(F_{t,i})$. The elimination rule is that the person with the lowest overall score is out. We define the consistency discriminant function:

$$L(\mathbf{F}_t) = \mathbb{I}\left\{\arg\min_i S_{t,i} = i_t^{\text{elim}}\right\} \tag{2}$$

Where i_t^{elim} is the actual eliminated player in history. Monte Carlo sampling: A large number of samples are generated from the uninformed prior $\mathbf{F}_t \sim \text{Dirichlet}(\mathbf{1})$, and only the samples that satisfy $L = 1$ are kept to form a feasible set. In order to characterize the time continuity supported by fans, a Bayesian sequential update is introduced: if the empirical mean of the feasible sample in week $t-1$ is $\overline{\mathbf{F}}_{t-1}$, then the Dirichlet priori parameter in week t is:

$$\boldsymbol{\alpha}_t = \lambda \overline{\mathbf{F}}_{t-1} + (1-\lambda)\mathbf{1} \tag{3}$$

Where λ is the momentum coefficient, which is optimized by grid search (the percentage system is optimal $\lambda = 49$, and the ranking system is $\lambda = 1.0$. For each player, the posterior mean $\mathbb{E}[F_{t,i}]$ and 95% confidence interval were calculated, and the weekly consistency acceptance rate $A_t = \dfrac{\text{number of feasible samples}}{\text{total number of samples}}$ was defined. The model finally reproduced the historical elimination with 99.68% accuracy.

Cause and effect: Since direct regression is not feasible due to the complete absence of fan data, generative constraint sampling is used and time smoothing is introduced in combination with Bayesian updates, so that the inversion distribution not only conforms to the hard constraint of elimination, but also reflects the inertia of fan support.

2.2. Fan-judge composite ranking model

This model aims to compare the relative influence of fans and judges under the ranking and percentage systems. First, based on the posterior mean of fan votes $\hat{F}_{t,i}$ obtained in section 2.1 and the known judges' scores, the weekly standardization is carried out. Define the ranked composite score:

$$C_{t,i}^{\text{rank}} = \text{rank}(J_{t,i}) + \text{rank}(\hat{F}_{t,i}) \tag{4}$$

Percentage system comprehensive score:

$$C_{t,i}^{\text{pct}} = \tilde{J}_{t,i} + \hat{F}_{t,i} \tag{5}$$

Calculate the Spearman correlation coefficient between the comprehensive ranking and the fan ranking and the judge ranking:

$$\rho_{\text{overall, fan}} = \frac{\text{cov}(\text{rank}(C_t), \text{rank}(\hat{F}_t))}{\sigma_{\text{rank}(C_t)}\sigma_{\text{rank}(\hat{F}_t)}} \tag{6}$$

Similar definition $\rho_{\text{overall, judge}}$. Introducing bias indicators:

$$\text{Bias}_{\text{fan}} = \text{rank}(C_t) - \text{rank}(\hat{F}_t) \tag{7}$$

For the new rules from Season 28 (Judges Help), identify the bottom two of each week:

$$\text{BottomTwo}_t = \{i : \text{rank}(J_{t,i}) \geq N_t - 1\} \tag{8}$$

Definition Whether to Eliminate by Judges:

$$\text{JudgeDecided}_t = \mathbb{I}\{\text{eliminated} \in \text{BottomTwo}_t\} \tag{9}$$

The direct elimination rate of judges was calculated, and the difference in elimination rate between the bottom group and the non-bottom group was compared. At the same time, track the weekly ranking evolution of typical players.

The aggregation method of the weights of fans and judges under different competition systems directly affects the elimination results, and the degree of reflection of the competition system on the wishes of fans can be quantified through the correlation coefficient and rescue rate, and case studies can visually show extreme situations.

2.3. Multidimensional impact model for rule stage perception

This model quantifies the impact of contestants' age, industry background, and professional dance partners on judges' scores and fan votes. The season is first divided into three stages: ranking system (S1-2), percentage system (S3-

27), and new rules (S28+). Two ridge regression models were established for each stage, and the dependent variables were standardized judge score $\tilde{J}_{it}$ and standardized fan vote $\tilde{F}_{it}$ (obtained from section 2.1). The independent variables include: age Age_i (continuous), industry background mute variable Industry_{ik} (such as singers, actors, etc.), professional dance partner mute variable Partner_{im} (reflecting professional ability), and competition progress Week_t as control variables. Model form:

$$\tilde{J}_{it} = \beta_0 + \beta_{\text{age}}\text{Age}_i + \sum_k \beta_{\text{ind},k}\text{Industry}_{ik} + \sum_m \beta_{\text{part},m}\text{Partner}_{im} + \beta_{\text{week}}\text{Week}_t + \varepsilon \tag{10}$$

The ridge regression estimation coefficient is used, and the loss function is $\| \mathbf{y} - \mathbf{X}\boldsymbol{\beta}\|^2 + \alpha\| \boldsymbol{\beta}\|^2$

α is selected by cross-validation. Since the dependent variable is standardized, the absolute value of the coefficient directly reflects the importance of the factor. Further define preference differences:

$$\Delta\beta_f = \beta_f^{\text{judge}} - \beta_f^{\text{fan}} \tag{11}$$

A positive value means that the judges value this factor more, and a negative value means that fans value it more.

Judges and fans may evaluate contestants based on different dimensions, and through the phased ridge regression, the confusion caused by rule changes can be stripped away, and the standardized coefficients can be directly compared to reveal the essential differences in the concerns of the two.

3. Results and Discussion

3.1. Reverse fan inferring results

Table 1. Model Consistency and Uncertainty Indicators (Quarterly Average)

Competition stage	Elimination reproduction accuracy	Fan vote average confidence interval width	Weekly consistency acceptance rate At
Ranking System (S1-2)	99.68%	7.697	0.32
Percentage System (S3-27)	99.68%	0.166	0.28
New Rules (S28+)	99.68%	0.171	0.30

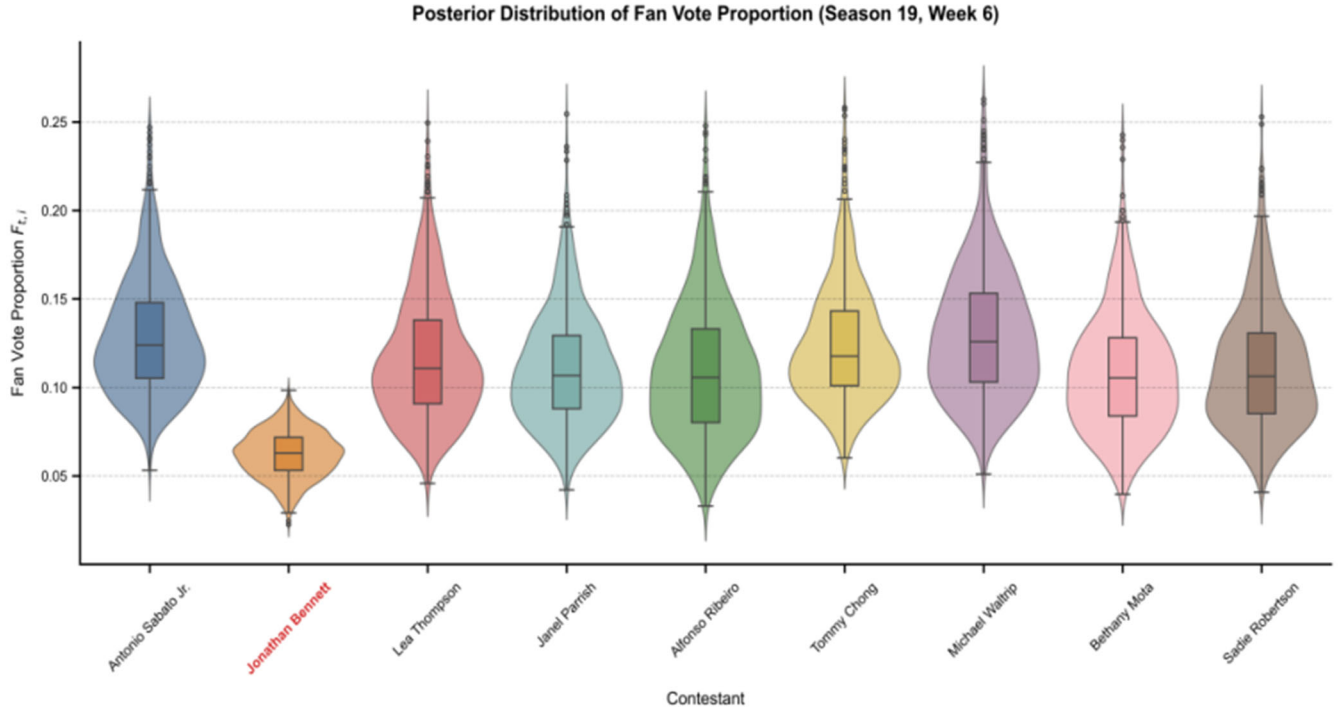

Figure 1. Example of a fan vote posterior distribution

Fig. 1 shows that the posterior distribution of fan votes of eliminated players has shifted significantly to the left, indicating that low fan support is the direct reason for elimination. Table 1 shows that the model reproduces historical elimination with 99.68% accuracy in all three formats, which verifies the effectiveness of inversion. The average confidence interval width is 0.166 under the percentage system and about 7.7 under the ranking system, reflecting that the estimation accuracy is acceptable but still fluctuates, especially in the middle and late seasons. This provides a credible estimate of fan votes for subsequent analysis.

3.2. Composite ranking system compares results

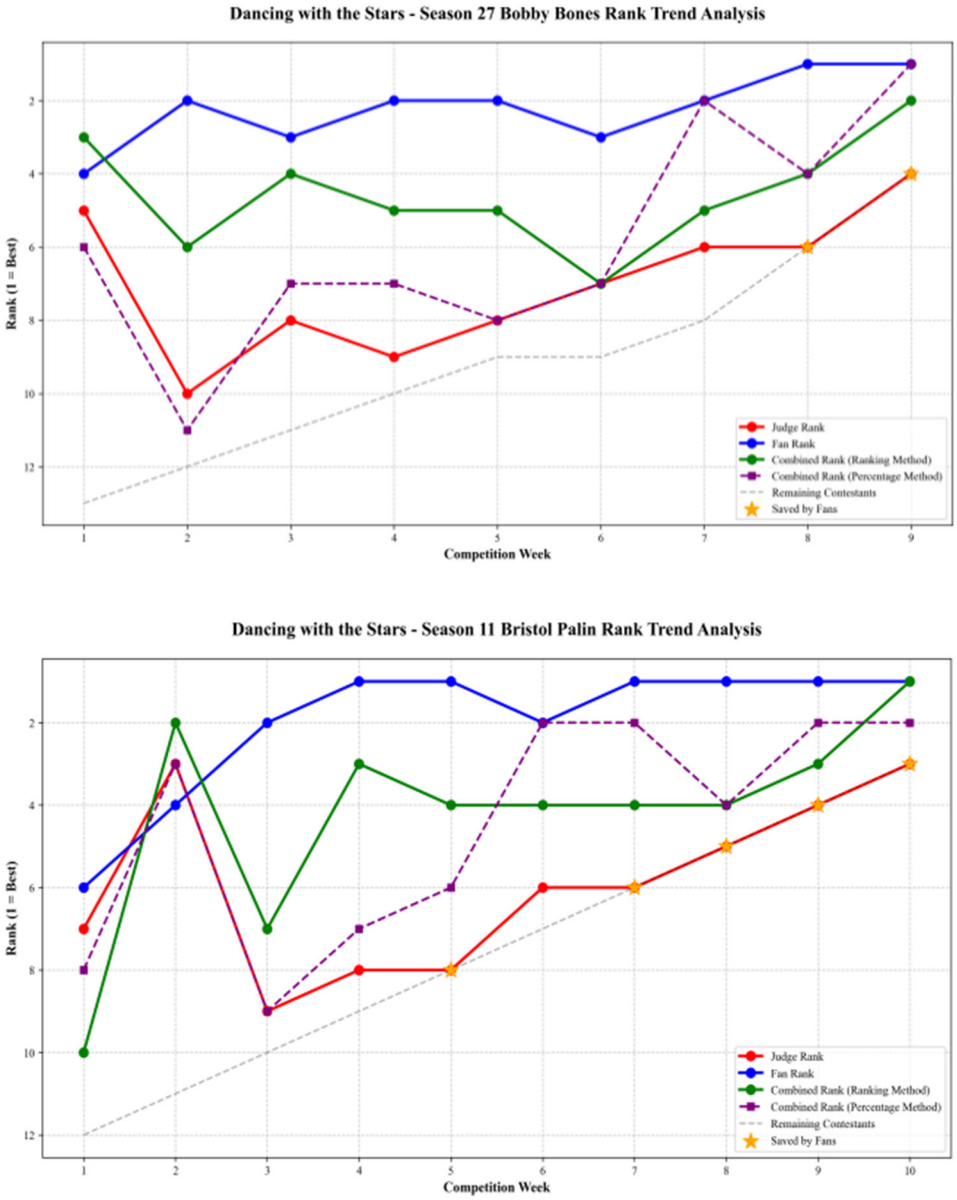

Figure 2. Bobby Bones (S27) vs Bristol Palin (S11) Weekly Ranking Evolution

Table 2. Comparison of influence under the ranking system and the percentage system

Methods	Comprehensive vs Fans ρ	General vs Judges ρ	Fans Save the Lowest Judge Probability	The Judge Saves the Lowest Fan Player Probability
Ranking system	0.88	0.86	60%	100%
Percentage system	0.12	0.73	51%	0%
New rules (S28+): 62.5% of judges are eliminated, 0.23 is eliminated by the bottom group, and 0.04 is not in the bottom group				

Fig. 2 shows that Bobby Bones reversed the judges' low score (multiple times at the bottom) with a very high fan ranking (always in the top 4) under the percentage system, and finally won the championship; Bristol Palin also relied on fan support to finish third. Table 2 quantitatively reveals the systematic differences: the influence of fans and judges is highly balanced under the ranking system (ρ=0.88, 0.86), and the probability of fans saving the lowest judges is 60%. Under the percentage system, the influence of fans is severely diluted (ρ=0.12), and the probability of saving the judges is 0. After the introduction of the new rules, the judges directly decided to eliminate 62.5%, and the elimination rate of the bottom group was 5.75 times that of the non-bottom group, proving that the intervention of the judges was significantly enhanced. These results are consistent with typical cases, indicating that the design of the competition system directly adjusts the weight of fans and judges.

3.3. Multidimensional factors affect and preference difference results

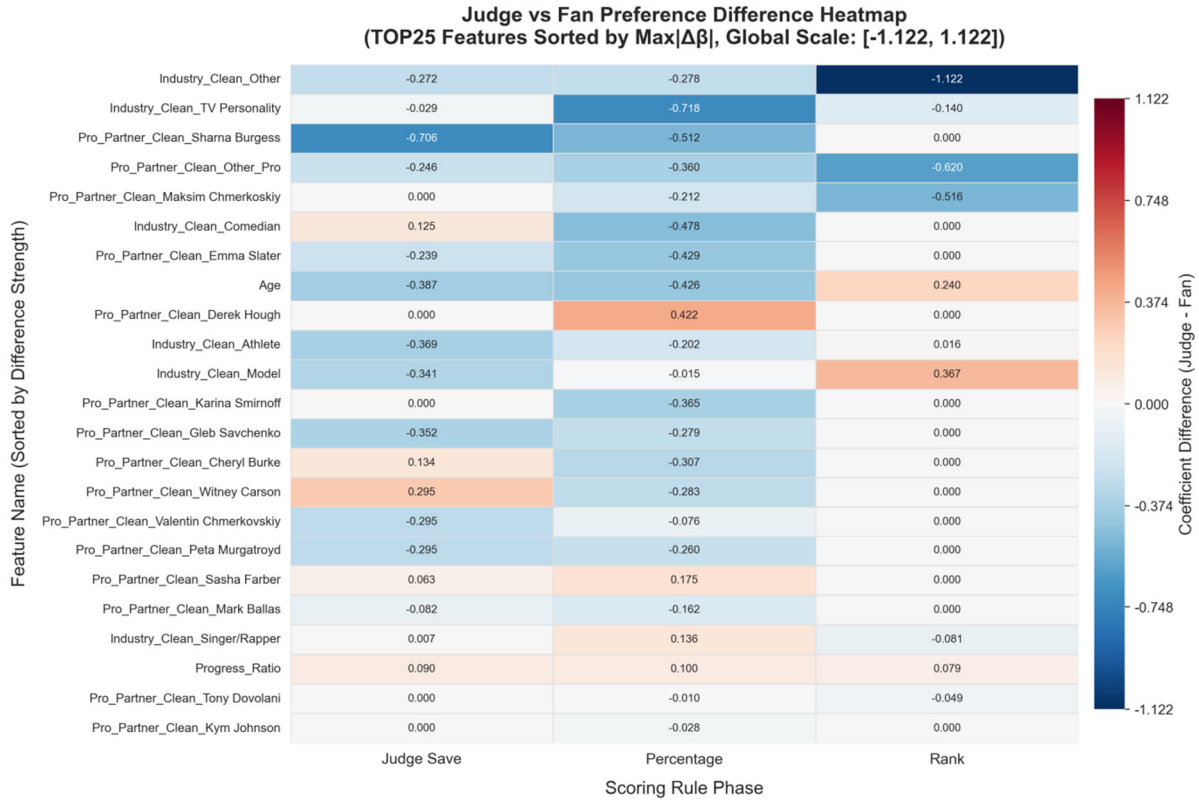

Figure 3. Feature importance heat map

Table 3. Key factors were standardized coefficients and differences in preferences

Rule stage	Goal	Age	Professional Dance Partner	Industry Background (Athlete)	Competition Process
Ranking system	Judges	0.07	0.81	0.40	0.12
Ranking system	fans	0.31	0.45	0.48	0.05
Δβ (Judge-Fans)	-	-0.24	+0.36	-0.08	+0.07
percentage system	Judges	0.41	0.18	0.32	0.21
percentage system	fans	0.01	0.16	0.10	0.03
Δβ	-	+0.40	+0.02	+0.22	+0.18
New rules	Judges	0.37	0.51	0.21	0.09
New rules	fans	0.01	0.36	0.16	0.02
Δβ	Goal	+0.36	+0.15	+0.05	+0.07

Fig. 3 and Table 3 reveal systematic differences: judges consistently prioritize professional partners (highest coefficient 0.81), while fans focus more on industry background (highest coefficient 0.48) and age (fans value age only under the ranking system, with a negative Δβ). Under the ranking system, Δβ for professional partners is +0.36, indicating judges' significant preference for technical skills; under the percentage system, Δβ for age is +0.40, showing that judges believe older age results in deductions while fans ignore it; under the new rules, the difference narrows slightly but persists. This confirms the core conclusion that judges adopt a "skill-oriented" approach while fans adopt an "identity-emotion-oriented" one, with the stage of rules influencing the intensity of preferences.

4. Conclusion

This paper proposes a three-step algorithm framework to solve the problems of complete lack of fan voting data in the "Dancing with the Stars" program and the difficulty of quantifying the interaction mechanism between judges and audiences. The first step is to successfully invert the posterior distribution of weekly fan votes through Bayesian sequential update and Monte Carlo constraint sampling, and the model has a reproducibility accuracy of up to 99.68% for historical elimination, and quantifies the uncertainty of estimation, providing reliable input for subsequent analysis. The second step is to construct a fan-judge composite ranking model, which systematically compares the relative influence of fans and judges under the ranking system and the percentage system, and finds that the ranking system can better balance

the weights of the two (correlation coefficient 0.88 vs 0.86), while the percentage system severely dilutes the influence of fans (correlation coefficient is only 0.12). At the same time, after the introduction of the judge's life-saving mechanism, the judges directly decide to eliminate 62.5%, and the elimination rate of the bottom group soars. The third step is to establish a multi-dimensional influence model perceived in the rule stage, using Ling regression to isolate the contributions of factors such as age, industry background, and professional dance partner, and reveal the fundamental difference between judges and fans through the preference difference Δβ: the judges always focus on technical factors (professional dance partner coefficient up to 0.81), and fans always focus on identity factors (industry background coefficient up to 0.48), and this difference intensifies with the evolution of the competition system.

The main contributions of this study are: (1) proposing a general method of hidden variable inversion that can be extended to other competitive programs containing hidden voting; (2) quantify the moderating effect of competition system design on fairness and entertainment, and provide data-driven rule optimization suggestions for program producers; (3) It reveals the structural differences between professional evaluation and public preference, and enriches the theoretical perspective of entertainment data analysis. Based on the research results, it is suggested that the program team should use dynamic weights to protect technical players in the early stage, and appropriately increase the weight of fans in the later stage to increase suspense. At the same time, the conflict intensity indicator can be introduced to adaptively adjust the judge-fan weight to achieve a balance between fairness and entertainment. Future work can consider external factors such as the dynamic progress curve of players and social media popularity to further refine the model.

The algorithm in this paper has been verified by 34 quarters of historical data and has practical deployment potential. All code and data (desensitized) are open source, making it easy for peers to reproduce and improve.

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